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KANTOWSKI–SACHS COSMOLOGICAL MODEL IN SAEZ–BALLESTER THEORY OF GRAVITATION

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Abstract: We have investigated Kantowski–Sachs cosmological model in the presence of perfect fluid distribution in a scalar-tensor theory of gravitation proposed by Saez and Ballester (Phys. Lett. A113, 1985, 467) with the aid to stiff fluid. Some physical and geometrical behaviour of the cosmological model are also studied.

Keywords : Kantowski-Sachs model, Saez-Ballester theory, perfect fluid, stiff fluid.

1. Introduction

In the last few decades a considerable interest has been focused by cosmologist in formation of alternative theories of gravitation. Brans and Dicke [2] is one of them who have attracted many researchers towards the scalar-tensor theories of gravitation. In this theory, the scalar field has the dimension of inverse of the gravitational constant G and its role is confined to its effects on gravitational field equations. Later Saez and Ballester [13] have developed a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field in a simple manner. In spite of dimensionless character of the scalar field, an antigravity regime appears. This theory suggests a possible way to solve the missing-matter problem in non-flat FRW models. A cosmological model with negative constant deceleration parameter in a scalar-tensor theory has studied by Reddy et al. [11]. Reddy et al. [12] have also studied axially symmetric cosmic strings in scalar-tensor theory of gravitation. Bianchi type-VI string cosmological models in Saez-Ballester's scalar- tensor theory of gravitation have investigated by Adhav et al. [1]. Shri Ram et al. [14] have investigated Bianchi type-V cosmological models with perfect fluid and heat flow in Saez-Ballester theory. LRS bianchi type-V cosmology with heat flow in scalar-tensor theory has obtained by Singh [15]. Bianchi type-V cosmological models with constant deceleration parameter within the framework of scalar-tensor theory of gravitation proposed by Saez and Ballester have investigated by Tiwari [16]. Kandalkar et al. [4] and Katore et al. [7] have studied Kantowski-Sachs domain wall and FRW cosmological solutions respectively in Saez-Ballester's scalar-tensor theory of gravitation.

Beside the Binachi type metrices, the Kantowski-Sachs [8] models are also describing spatially homogeneous universes. Kantowski-Sachs viscous fluid cosmological model with a varying cosmological constant has discussed by Kandalkar et al. [5]. Kandlarkar et al. [6] have also obtained string cosmology in Kantowski-Sachs space-time with bulk viscosity and magnetic field. Rao and Neelima [10] have studied Kantowski-Sachs string cosmological model with bulk viscosity in general scalar tensor theory of gravitation. Pawar and Dagwal [9] have studied Kantowski-Sachs cosmological models in scalar-tensor theory of gravitation. Recently, David and Gernot [3] have investigated Kantowski-Sachs cosmology with Vlasov matter.

The purpose of the present work is to obtain Kantowski-Sachs cosmological model in a scalar-tensor theory of gravitation proposed by Saez and Ballester in the presence of a perfect fluid by using stiff fluid. This paper is organized as follows: The metric and field equations are presented in Section 2. Section 3 deals with solution of the field equations. Some physical and geometrical properties of the model are presented in Section 4. Finally the conclusions of the paper are presented in Section 5.

2. The Metric and Field Equations

We consider the Kantowski–Sachs space time metric in the form

$$ds^2 = dt^2 - A^2 dr^2 - B^2 (d\theta^2 + \sin^2\theta\, d\phi^2), \tag{1}$$

where the metric potentials A and B are functions of cosmic time t alone.

The field equations given by Saez and Ballester [13] for the combined scalar and tensor fields are

$$G_{ij} - \omega\varphi^n (\varphi_{,i}\, \varphi_{,j} - \frac{1}{2} g_{ij}\, \varphi_{,k}\, \varphi^{,k}) = - T_{ij} ,$$

(2)

where the scalar field φ satisfies the equation

$$2\varphi^n \varphi^{,i}_{;i} + n\varphi^{n-1} \varphi_{,k}\, \varphi^{,k} = 0 \tag{3}$$

and $G_{ij} = R_{ij} - \frac{1}{2} R\, g_{ij}$ is the Einstein Tensor, ω is a dimensionless coupling constant, n is an arbitrary constant and T_{ij} is the stress-energy tensor of the matter. Comma and semi-colon respectively denote partial and covariant derivative with respect to cosmic time.

The energy–momentum tensor of a perfect fluid has the form.

$$T_{ij} = (p+\rho)\, u_i\, u_j - p g_{ij}, \tag{4}$$

where p is the thermodynamical pressure, ρ the energy density, u_i the four-velocity of the fluid satisfying.

$$g_{ij}\, u_i\, u^j = 1. \tag{5}$$

In co-moving system of coordinates, we have $u_i = (0, 0, 0, 1)$. For the energy momentum tensor (4) and Kantowski–Sachs space time (1), Einstein's field equations (2) in Saez-Ballester theory, lead to the following set of independent differential equations.

$$\frac{2\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{1}{B^2} = -p + \frac{1}{2}\omega\varphi^n\dot{\varphi}^2 , \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -p + \frac{1}{2}\omega\varphi^n\dot{\varphi}^2 , \tag{7}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{1}{B^2} = \rho - \frac{1}{2}\omega\varphi^n\dot{\varphi}^2 , \tag{8}$$

$$\ddot{\varphi} + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{\varphi} + \frac{n}{2}\frac{\dot{\varphi}^2}{\varphi} = 0 , \tag{9}$$

where an overdot stands for the first and a double overdot for second derivative with respect to t.

3. Solution of the Field Equations

The field equations (6) – (9) are a system of four equations with six unknown parameters A, B, p, ρ, φ and ω. Two additional constraints relating these parameters are required to obtain explicit solutions of the system. We first assume that the shear σ is proportional to the expansion θ. This condition leads to the following relation between the metric potentials:

$$A = B^m, \qquad (10)$$

where m is a positive constant.

Secondly, stiff fluid can be regarded as perfect fluid having the energy-momentum tensor given by (4) characterized by the equation of state

$$p = \rho. \qquad (11)$$

Equations (6) and (7) lead to

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \left(\frac{\dot{B}}{B}\right)^2 - \frac{1}{B^2} = 0. \qquad (12)$$

Using Equations (7) and (8) in Equation (11), we get

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 3\frac{\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{1}{B^2} = 0. \qquad (13)$$

On adding Equations (12) and (13), we obtain

$$\frac{\ddot{A}}{\dot{A}} + 2\frac{\dot{B}}{B} = 0. \qquad (14)$$

This after integration leads to

$$\dot{A}B^2 = k_1, \qquad (15)$$

where k_1 is a constant of integration.

Using Equation (10) in Equation (15), we get

$$A^{\frac{2}{m}}\dot{A} = k_1. \qquad (16)$$

Integrating (16), we obtain

$$A = L\,(k_1 t + k_2)^{\frac{m}{m+2}}, \qquad (17)$$

where $L = \left(\frac{m+2}{m}\right)^{\frac{m}{m+2}}$ and k_2 is a constant of integration.

Using Equation (17) in Equation (10), we get

$$B = M\,(k_1 t + k_2)^{\frac{1}{m+2}}, \qquad (18)$$

where $M = L^{\frac{1}{m}}$.

Hence the metric (1) reduces to the form

$$ds^2 = \frac{dT^2}{k_1^2} - L^2\, T^{\frac{2m}{m+2}}\, dr^2 - M^2 T^{\frac{2}{m+2}}(d\theta^2 + \sin^2\theta\, d\phi^2), \qquad (19)$$

where $k_1 t + k_2 = T$.

In this case the expression for the scalar field φ, from Equation (9), is found as

$$\varphi = \left[\frac{(n+2)}{2}\left\{\frac{k_3}{k_1}\log T + k_4\right\}\right]^{\frac{2}{n+2}},$$

(20)

where k_3 and k_4 are constants of integration.

4. Some physical and geometrical properties

The expressions for the scalar of expansion θ, magnitude of shear σ^2, the average anisotropy parameter A_m , deceleration parameter q and spatial volume V for the model (19) are given by

$$\theta = \frac{k_1}{T}, \tag{21}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{m-1}{m+2}\right)^2 \frac{k_1^2}{T^2}, \tag{22}$$

$$A_m = 2\left(\frac{m-1}{m+2}\right)^2, \tag{23}$$

$$q = 2, \tag{24}$$

$$V = K\,T^2 \sin\theta, \tag{25}$$

where $K = LM^2$

The directional Hubble's parameters $H_1 = H_r$, $H_2 = H_\theta$, and $H_3 = H_\phi$ $(H_2 = H_3)$ are given by

$$H_1 = \left(\frac{m}{m+2}\right)\frac{k_1}{T}, \tag{26}$$

$$H_2 = H_3 = \frac{1}{(m+2)} \frac{k_1}{T}. \tag{27}$$

The generalized mean Hubble's parameter H is given by

$$H = \frac{k_1}{3T}. \tag{28}$$

Curvature scalar R is calculated and given as

$$R = 2\left[\frac{1}{M^2 T^{\frac{2}{m+2}}} - \frac{2m}{(m+2)^2}\frac{k_1^2}{T^2}\right]. \tag{29}$$

The energy density ρ and the thermodynamical pressureare p are given by

$$\rho = p = \frac{(2m+1)}{(m+2)^2}\frac{k_1^2}{T^2} + \frac{1}{2}\omega\frac{k_3^2}{T^2} - \frac{1}{M^2\ T^{\frac{2}{m+2}}}. \tag{30}$$

From the above results, it can be seen that the spatial volume is zero at T = 0. This shows that the universe starts evolving with zero volume at T = 0 and expands with cosmic time T. The expansion scalar, shear scalar, energy density and the pressure are infinite at T = 0, which clearly indicate the point-type singularity at the initial epoch. The directional Hubble's parameters and the generalized mean Hubble's parameter both are infinite at this singularity point. The scalar curvature R tends to infinity at the point of singularity. The scalar field function (φ) is an increasing function of T and it tends to infinity when $T \rightarrow \infty$. The expansion scalar, shear scalar and all the directional Hubble's parameter tend to zero as T $\rightarrow \infty$. We also see that R $\rightarrow$ 0 as T $\rightarrow \infty$. This shows that our space-time is flat for large values of T. The mean anisotropy parameter is constant throughout the evolution of the universe. The positive values of the deceleration parameter indicates that the model decelerates in the standard way. Since $\lim_{T \to \infty} \frac{\sigma^2}{\theta^2} \neq 0$ and it is independent of cosmic time. Hence our model does not approach the isotropy for large value of T.

5. Conclusion

In this paper we have considered Kantowski–Sachs cosmological model in Saez–Ballester scalar tensor theory of gravitation in the presence of perfect fluid. Exact solutions of Einstein's field equations for this model have been obtained by using two conditions $A = B^m$ and $p = \rho$. The model has point type big-bang singularity at T = 0 and it is expanding and decelerates in the standard way. The model is flat and does not approach the isotropy for large value of T. We have also discussed some physical and geometrical properties of the cosmological model.

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